

IRSTI 44.29.37

## **AUTO-OSCILLATIONS OF WIRES OF HIGH-VOLTAGE POWER LINES (ANCHOR SPAN)**

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*Dangerous rapprochement or whipping of wires in flight can be caused by dancing the wires. There fore, the distance between the wires and cables should be selected taking into account the expected intensities of the dancing wires. The purpose of this article is to determine the possible intense dancing of the split phase wires based on the self-oscillatory regime of the icy wires of the split phase at the anchor span. Investigations of the self-oscillating process were carried out by the Van Der Pol method. The research results can be used in the design of high-voltage power lines, studies of the phenomena of dancing and in the development of measures to protect lines from dancing wires.*

**Keywords:** power line, wire dance, self-oscillations, equation of motion, degree of freedom, wind speed, dance intensity, stability.

## **ЖОҒАРЫ КЕРНЕУЛІ ӘУЕ ЖЕЛІЛЕРІ СЫМДАРЫНЫҢ АВТОТЕРБЕЛІСТЕРІ (АНКЕРЛІ ПРОЛЕТ)**

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*Сымдардың қауіпті жақындасуы немесе аралықтағы сымдардың бұралуы олардың билеуіне әкеледі. Сондықтан, сымның өзара қашықтығын және де сымдар мен тростар арасындағы қашықтықты, билейтін сымдардың қарқындылығын ескере отырып таңдау керек. Осы мақаланың мақсаты – автотербелістік режим негізіндегі тарамдалған фазасымдарының анкерлік өткінде мүмкін болатын қарқынды билеуін анықтау болып табылады. Автотербелістік процесті зерттеу Ван – Дер - Пол әдісімен жүргізілді. Зерттеу нәтижелері жоғары вольтты электр желілерін жобалауда, сымдардың билеу құбылыстарын зерттеуде, және де желілерді сымдардың билеуінен қорғауға арналған шараларды жасауда қолдануға болады.*

**Негізгі сөздер:** әуе желілері, мұзды сымдардың билеулері, автотербеліс, математикалық модель, еркіндік дәрежесі, жел жылдамдығы, билеу қарқыны, орнықтылық.

## **АВТОКОЛЕБАНИЯ ПРОВОДОВ ВЫСОКОВОЛЬТНЫХ ЛИНИЙ ЭЛЕКТРОПЕРЕДАЧИ (АНКЕРНЫЙ ПРОЛЕТ)**

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*Опасные сближения или схлестывания проводов в пролете могут быть вызваны пляской проводов. Поэтому, расстояния между проводами, а также между проводами и тросами должны выбираться с учетом предполагаемой интенсивности пляски проводов. Цель настоящей статьи - на основе автоколебательного режима обледенелых проводов расщепленной фазы на анкерном пролете определение возможной интенсивной пляски проводов расщепленной фазы. Исследования автоколебательного процесса осуществлялись методом Ван-Дер-Поля. Результаты исследований могут быть использованы при проектировании*

*высоковольтных линий электропередач, исследований явлений пляски и при разработке мероприятий по защите линий от пляски проводов.*

**Ключевые слова:** линия электропередачи, пляска проводов, автоколебания, уравнение движение, степень свободы, скорость ветра, интенсивность пляски, устойчивость.

### **Introduction**

Dance is one of the most dangerous varieties caused by the wind of vibrations of icy wires of overhead lines (VL) [1]. When operating overhead lines for areas with frequent dancing of wires, it is necessary to provide measures to prevent overlapping of wires by increasing the distance between the wires (wires and cables), taking into account possible trajectories of movement of wires during dancing.

A number of works [1–11] have been devoted to assessing the possible intensities of dancing (the range of oscillations) depending on the parameters of the lines and weather conditions. These works are based on the analysis of long-term observational data on wire dancing on active lines and differ in the nature of the formulation of research problems and methods for solving them.

According to [2], the maximum range of dance  $Y_{\max}$  is estimated by the formula

$$Y_{\max} = \frac{0,26 \cdot V_w}{f}$$

where  $V_w$  is the wind speed,  $f$  is the transverse frequency of the wire (Hz).

According to [7.8], the parameters that make it most likely to determine the predisposition of the air line to the dance, as well as the possible amplitudes of the dance, are the parameter  $M'$

$$M' = 10,67 \frac{f^3}{\lambda_T \ell^2}$$

where  $f$  is the arrow of the wire sag,  $\lambda_T$  is the length of the supporting string of insulators,  $\ell$  - is the span.

According to the methodology, the amplitude of the dance is determined on the basis of a specially constructed nomogram. This technique applies to intermediate spans of overhead lines 110 and 220 kV with single wires.

The work [9] is devoted to assessing the maximum range of dancing of single wires and the split phase. On the basis of processing 166 observation data and additional experiments, the dependence of the maximum dance swing on the

diameter and arrow of the wire sag was obtained with some restrictions on the span ( $30 \leq \ell \leq 500$ ) and wind speed (for single wires  $V \leq 15$  and for split phases  $V \leq 10$ ):

For single wires

$$\frac{A_{pk-pk}}{d} = 80 \ln \frac{8f}{50d}$$

For split phases

$$\frac{A_{pk-pk}}{d} = 170 \ln \frac{8f}{500d}$$

where  $A_{pk-pk}$  is the swing of the dance,  $d$  - is the diameter of the wire,  $f$  - is the arrow of the sag of the wire at  $0^\circ\text{C}$ .

Similar works [10, 11] are devoted to estimating the maximum intensities of single-wave dancing based on the processing of long-term observational data. In these studies, when determining the maximum intensity of dancing, the main factor is not taken into account - the dependence of the intensities on the wind speed.

### **Objects and methods of research**

The object of the study is high-voltage power lines (power lines). The subject of the study is the dance of wires of power lines on the anchor span.

The purpose of this article is based on the study of the self-oscillatory regime of the icy wires of the split phase (RF) on the anchor span, to determine the possible intensities of dancing of the wires of the split phase.

Research of the self-oscillating process is carried out by the van der Pol method.

### **Results and their discussion**

With moderate winds, as a rule, the dance of the RF wires is characterized by insignificant (sometimes complete absence) torsional movements. For such cases, the mutual influence of torsional and linear (transverse) movements of the RF during dancing can be neglected and the oscillatory process without torsional movements can be considered.

In [12], a mathematical model of the wire dance of the RF power lines was obtained. The model takes into account two degrees of freedom - linear and torsional motion and is designed for the case when the points of attachment of the

wires to the supports are fixed (anchor span). If we exclude in this model the generalized coordinate of torsional motions, the initial

nonlinear system can be approximately reduced to a single equation with respect to linear displacement

$$\ddot{a}(t) + k_1 \dot{a}(t) + k_2 \dot{a}^3(t) + \omega_a^2 a(t) + k_3 a^2(t) + k_4 a^3(t) + k_{13} = 0 \quad (1)$$

where the coefficients of equation (1) are determined by the following expressions:

$$P_{sep} = 1,1P_0; \xi_0 = \frac{\rho d_{\Pi} C_{D0}}{2P_{sep}}; \xi = \xi_0 V^2; b_1 = \frac{gEF}{T\ell^2}; b_2 = \frac{gEF}{P_{sep}\ell^4}; b_3 = \frac{g\rho d_{\Pi}}{P_{sep}}; b_4 = \frac{4b_3}{\pi};$$

$$b_5 = \frac{b_4 d_{\Pi}}{R^2}; \omega^2 = \frac{\pi^2 gT}{2P_{sep}\ell^2} \left( 1 + \frac{EFP_{sep}^2 \ell^2}{2\pi^2 T^3} \right); k_1 = \frac{\delta \omega_a}{\pi} - \frac{b_4(C_{D0} + C_{L0})}{2} V; k_2 = \frac{b_4}{3V} C_{L1};$$

$$k_3 = 3\pi b_1; k_4 = \frac{\pi^4 b_2}{4}; k_{13} = b_3 \xi_0 C_{D0} V^4; \omega_a^2 = \frac{\pi^2 gT}{P_{sep}\ell^2} \left( 1 + \frac{8EFP_{sep}^2 \ell^2}{\pi^4 T^3} \right)$$

Here  $a(t)$  – the generalized coordinate of linear displacement,  $\rho$  – air density,  $d_{\Pi}$  – characteristic profile size,  $g$  – gravity acceleration,  $V$  – wind speed,  $\ell$  – span,  $E$  – elastic modulus,  $F$  – wire cross-sectional area,  $T$  – wire tension,  $P_0$  – weight of a unit length of wire,  $P_{sep}$  – weight units of wire length taking into account ice,  $\delta$  – damping decrement,  $R$  – splitting radius,  $C_{L0}$  and  $C_{D0}$  are stationary

aerodynamic characteristics coefficients.

According to [13]:  $C_{D0} = 1$ ;  $C_{L0} = 4$ ;  $C_{L1} = 12$ ;

To study the dance of wires of the RF with one degree of freedom, we write the nonlinear equation (1) in the normal Cauchy form. Introducing the notation  $a(t) = y_0$ ;  $\dot{a}(t) = y_1$ , we obtain

$$\dot{y}_0 = y_1$$

$$\dot{y}_1 = -\omega_a^2 y_0 + k_1 f(y_0, y_1, t) \quad (2)$$

Where

$$f(y_0, y_1) = -\left( y_1 + \frac{k_2}{k_1} y_1^3 + \frac{k_3}{k_1} y_0^2 + \frac{k_4}{k_1} y_0^3 + \frac{k_{13}}{k_1} \right)$$

To solve the nonlinear equation, we use the approximate Van der Pol method [14]. According to the van der Pol method, we will seek solutions (3) in the form

$$y_0(t) = A(t) \cos[\omega_a t + \psi(t)] \quad (3)$$

$$y_1(t) = -\omega_a A(t) \sin[\omega_a t + \psi(t)] \quad (4)$$

Where  $A(t)$  is the average amplitude of the dance (unknown function of time),  $\psi(t)$  is the variable initial phase.

The average amplitude of the dance characterizes the arithmetic mean of the values of greater and lesser amplitude

where  $A_B$  is the large amplitude of the dance (moving the wire from static equilibrium to the highest position),  $A_H$  is the smaller amplitude of the dance (moving the wire from static equilibrium to the lower position),  $\Sigma A$  is the intensity of the dance of the wires

The intensity in the stationary mode of dancing is determined by twice the average amplitude.

$$\Sigma A = 2A(t) = 2A \quad (6)$$

$$F_1(A) = -\frac{1}{2\pi\omega_a} \int_0^{2\pi} f(y_0, y_1) \sin \beta d\beta = -\frac{1}{2} A \left( 1 + \frac{3k_2\omega_a^2}{4k_1} A^2 \right) \quad (7)$$

$$F_2(A) = -\frac{1}{2\pi\omega_a A} \int_0^{2\pi} f(y_0, y_1) \cos \beta d\beta = \frac{3k_4}{8k_1\omega_a} A^2 \quad (8)$$

where  $\beta = \omega_a t + \psi$

When calculating integrals (7) and (8), unknown functions of time and are considered constant  $A(t) = A_0$  and  $\psi(t) = \psi$ . We add the calculated values of the integrals to the shortened van der Pol equations

$$\frac{dA}{dt} = k_1 F_1(A) = -\frac{k_1}{2} A \left( 1 + \frac{3k_2 \omega_a^2}{4k_1} A^2 \right) \quad (9)$$

$$\frac{d\psi}{dt} = k_1 F_2(A) = \frac{3k_4}{8\omega_a} A^2 \quad (10)$$

For the solution of (9), we explicitly multiply both sides of the equality by  $2A$

$$2A \frac{dA}{dt} = -k_1 A^2 \left( 1 + \frac{3k_2 \omega_a^2}{4k_1} A^2 \right)$$

$$\ln \left| \frac{1}{A^2} + \frac{3k_2 \omega_a^2}{4k_1} \right| = k_1 t + C \Rightarrow \frac{1}{A^2} = C e^{k_1 t} - \frac{3k_2 \omega_a^2}{4k_1}$$

The constant integration of  $C$  under the initial conditions  $t=0$ ,  $A(0)=A_0$  ( $A_0$  is the initial deviation) has the value

$$C = \frac{1}{A_0^2} + \frac{3k_2 \omega_a^2}{4k_1}$$

$$A = \pm \sqrt{\frac{4k_1 A_0^2}{(4k_1 + 3k_2 \omega_a^2 A_0^2) e^{k_1 t} - 3k_2 \omega_a^2 A_0^2}} \quad (11)$$

As  $t \rightarrow \infty$ , the average amplitude of the wire dance tends to a constant value. Below, the transformations take into account the condition

$$\lim_{t \rightarrow \infty} \sqrt{\frac{4k_1 A_0^2}{(4k_1 + 3k_2 \omega_a^2 A_0^2) e^{k_1 t} - 3k_2 \omega_a^2 A_0^2}} = \sqrt{\frac{2V^2}{\omega_a^2} \left( \frac{C_{D0} + C_{L0}}{C_{L1}} \right) \left( 1 - \frac{V_{kp}^H}{V} \right)}$$

As follows from the last expression, no matter how small (much) the initial deviation  $A_0$ , is, the amplitude of the dance over time will still monotonously approach a stationary value

$$\Sigma A = 2 \sqrt{\frac{2V^2}{\omega_a^2} \left( \frac{C_{D0} + C_{L0}}{C_{L1}} \right) \left( 1 - \frac{V_{kp}^H}{V} \right)} \quad (12)$$

Where  $V_{kp}^H$  – the lower critical speed at which the wire dance is excited is determined by the formula

$$V_{kp}^H = \frac{\delta \omega_a P_{sep}}{2g\rho d_{\Pi} (C_{D0} + C_{L0})} \quad (13)$$

Make a replacement  
 $z = \frac{1}{A^2} \Rightarrow 2A \frac{dA}{dt} = -\frac{1}{z^2} \frac{dz}{dt}$

and transform the equation taking into account the replacement

$$\frac{d \left( z + \frac{3k_2 \omega_a^2}{4k_1} \right)}{z + \frac{3k_2 \omega_a^2}{4k_1}} = k_1 dt$$

The solution to the last equation has the form (in the final result, the substitution  $z=1/A^2$  is taken into account)

Taking into account constant integration, solution (9) has the form

$k_1 < 0$  at  $V > V_{kp}^H$ , where  $V_{kp}^H$  – the lower critical speed at which the wire dance is excited

(independent of the initial deviation). Thus, the intensity of dancing in a stationary mode according to (6) is determined by the expression

We study the oscillatory process for stability at the equilibrium point [14]. The state of equilibrium is determined based on the condition  $F_1(A)=0$ . According to formula (7), we have two equilibrium states:

$$A_1 = 0 \quad \text{and} \quad A_2 = \sqrt{-\frac{4k_1}{3k_2\omega_a^2}} = \sqrt{\frac{3V[2\delta\omega_a - \pi b_4(C_{D0} + C_{L0})V]}{2\pi b_4 C_{L1}}} \quad (14)$$

Define the derived function  $F_1(A)$

$$\frac{dF_1(A)}{dA} = -\frac{1}{2} \left( 1 + \frac{9k_2\omega_a^2}{4k_1} A^2 \right) \quad (15)$$

$$\left. \frac{dF_1(A)}{dA} \right|_{A=A_2} = -\frac{1}{16\pi b_4 C_{L1}} \left\{ 8\pi b_4 C_{L1} + \frac{27k_2}{k_1} \omega_a^2 V [2\delta\omega_a - \pi b_4 V (C_{D0} + C_{L0})] \right\}$$

From the last expression it follows that the equilibrium state  $A_2$  is stable only under the condition

$$2\delta\omega_a - \pi b_4 V (C_{D0} + C_{L0}) \geq 0$$

whence follows the value of the minimum wind speed at which the oscillatory process does not develop (formula 13)

$$V \leq \frac{\delta\omega_a P_{sep}}{2g\rho d_{\Pi} (C_{D0} + C_{L0})}$$

Below are the comparisons of the calculation results performed according to formula (12) with the results of theoretical calculations at various wind speeds (Figure 1). Theoretical data are obtained on the basis of modeling a mathematical model in a Mathcad environment using the Runge-Kutta method.

The calculations were performed for wire grade ASO - 330/39 with the following characteristics: Young's modulus  $E = 7700 \text{ DaN / mm}^2$ ; wire diameter  $d_P = 24 \text{ mm}$ ; the cross-sectional area of the wire  $F = 339.6 \text{ mm}^2$ ; weight unit length of wire  $R_{ver} = 1,132 \text{ daN / m}$ . Characteristics of the lines: The split phase consists of 3 wires ( $n = 3$ ), the splitting radius is  $R = 0.23 \text{ m}$ . In the calculation, the air density is taken to be  $\rho = 0.11 \text{ daN * s}^2 / \text{m}^4$  and the attenuation decrement is  $\delta = 0, 12$ .

As the comparison results show, the calculation formula (12) is applicable in a limited range of speeds, the values of which depend on the span. So, for  $\ell = 200 \text{ m}$ , the calculation

By supplying  $A_1$  and  $A_2$  the values and alternately in the expression (15), we have the stability condition

$$\left. \frac{dF_1(A)}{dA} \right|_{A=A_1} = -\frac{1}{2} < 0 \quad (\text{equilibrium steady})$$

formula is valid at a wind speed not exceeding  $12 \text{ m / s}$ , beyond which the character of the dependence does not coincide with the simulation results. Similarly, for  $\ell = 300 \text{ m} - 8 \text{ m / s}$  and for  $\ell = 400 \text{ m} - 7 \text{ m / s}$ . Such limitations are due to the fact that when deriving the calculation formula, the influence of torsional movements on the nature of the dance was not taken into account. At moderate wind speeds, the influence of torsional movements on the nature of the dance is negligible and can be neglected to some extent. However, this assumption is unacceptable at high wind speeds. As the simulation results show, at high wind speeds there is an abrupt decrease in the intensity of dancing (Figure 1) and, on the contrary, an abrupt increase in the intensity of torsional vibrations, that is, an energy exchange occurs between linear and torsional vibrations.

In general, in the allowed range of wind speeds, the discrepancy between the data calculated according to formula (12) and the theoretical data is insignificant. The maximum discrepancy is observed at  $\ell = 300 \text{ m}$  and  $V = 8 \text{ m / s}$ , which does not exceed 15%. Thus, the calculation formula (12) can be used to assess the intensity of wire dancing in a certain range of wind speeds. The allowed range of wind speeds can be set based on an analysis of the original mathematical model of wire dancing (with two degrees of freedom).

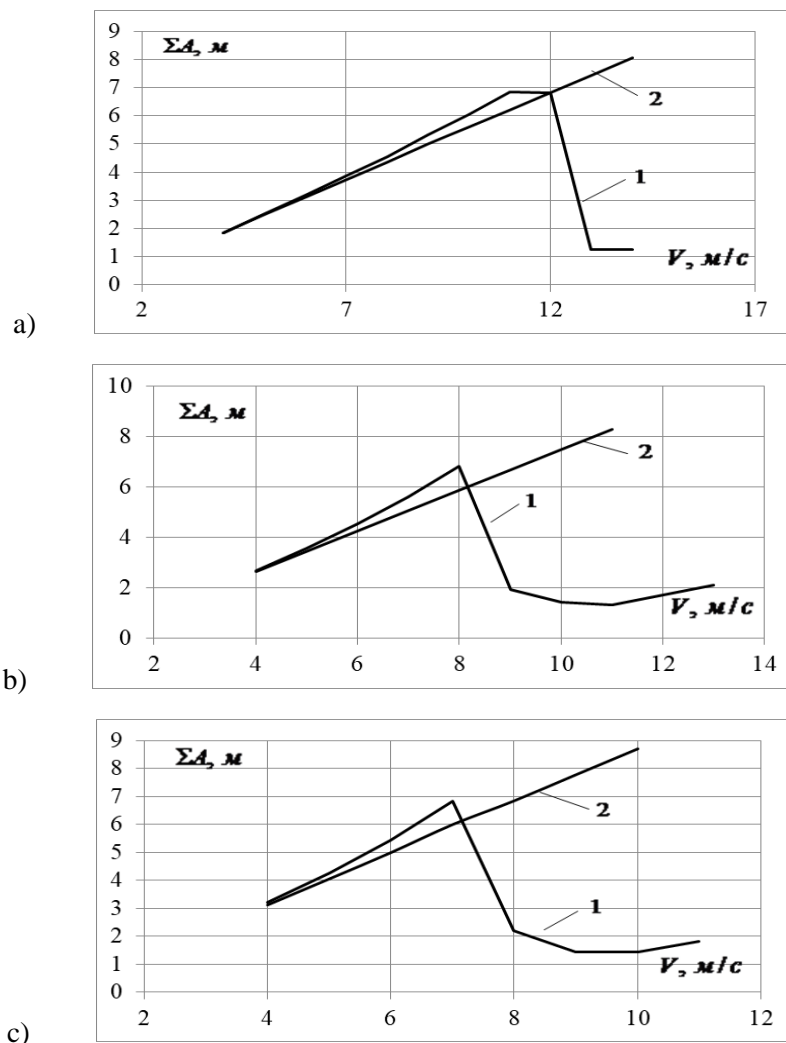


Figure 1. The dependence of the intensity of the dancing wires of the RF on wind speed.  $\sigma = 10 \text{ daH} / \text{mm}^2$ . A) -  $\ell = 200$  m B) -  $\ell = 300$  m C) -  $\ell = 400$  m.

1 - according to the mathematical model of the wire dance of the R

### Conclusion

1. A calculation formula has been obtained for determining the intensities of one-half-wave dancing of RF wires at given wind speeds, known power line parameters and aerodynamic characteristics of icy wires. 2 - according to the calculation formula (12).

2. The calculation formula is applicable in a limited range of wind speeds. The permissible speed range can be determined (in the future) based on the analysis of the initial mathematical model of dancing, taking into account the mutual effects of linear and torsional vibrations in the process of dancing.

3. The developed technique allows you to determine the sum of the amplitudes ( $A_B + A_H$ ), but does not provide information on the ratio of amplitudes ( $A_B / A_H$ ), which is its disadvantage.

4. The results of the study can be used in the design of high-voltage power lines, the study of the phenomena of dance and in the development of measures to protect overhead lines from dancing wires.

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UDC 550.3  
IRSTI 37.31

## AXISYMMETRIC BEND OF THE LITHOSPHERIC PLATE OF THE EXPONENTIAL PROFILE

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*Earth 's seismic and volcanic activity is related to plate tectonics. Thin elastic surface plates form a lithosphere which experience different loads. The article considers a new model of the stress-strain state of the axisymmetric lithospheric plate of exponential profile in a non-uniform temperature field and under the influence of transverse forces. Novelty of solution of this problem lies in study by method of partial sampling of nonlinear differential equation with non-uniform coefficients when the lithospheric plate is bent. There are obtained regularities of change of radial force and bending moments under action of radial uniformly distributed load and volumetric centrifugal forces, as well as a result of temperature heating. A graphic analysis indicates the non-linear nature of their distribution, which significantly affects the shape of a curved plate.*

**Key words:** transverse forces, temperature field, lithospheric plate, exponential profile, axisymmetric bend.